



1. Let S be the set of all integers of the form $Pn^2 + 1$ where P is a prime number greater than 5. Let N be the largest integer that divides every member of S . Find, with proof, the value of N .
2. In triangle ABC , $(\cos B)(\cos C) = \cos A$. Find, with proof, the numerical value of $(\tan B)(\tan C)$.
3. Suppose that $n + 1$ boys are lined up shoulder-to-shoulder from left to right in a straight line. Prove that it is always possible to select $n + 1$ boys to take one step forward so that, going from left to right, their heights are either consistently increasing or consistently decreasing.
4. The lengths of the sides of triangle ABC are in the ratio of 4:5:6. The bisector of the largest angle of the triangle is drawn. Prove that one of the two triangles formed also has sides whose lengths are in the ratio of 4:5:6.
5. All the factors of the polynomial $P(x) = ax^3 + bx^2 + cx + d$ are linear with integer coefficients, and neither a nor d are zero or one. Find all possible pairs (a, d) , and prove that you have found them all.

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