THE 2021 2022 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box is marked. Zero points are given for an incorrect answer or if multiple boxes are marked. Note that wild guessing is likely to lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

NO CALCULATORS

1. Consider the following 5 statements:

   (i) Statement (ii) is true.
   (ii) At most one of these 5 statements is true.
   (iii) All 5 of these statements are true.
   (iv)
   (v)

   The last two statements are invisible. Which of the following is correct?

   (A) Only statement (i) is true. (B) Only statement (ii) is true.

   (C) Only one of statements (iv) or (v) is true. (D) Both statements (iv) and (v) are true.

   (E) Neither of statements (iv) and (v) are true.

2. Compute the sum of all values of >, u Q > Q s r, such that the base 10 representation of t s r s$_o$ is the square of an integer.

   (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

6. If $0 < \theta < 90$, and $15\cos\theta - 20\sin\theta = 7$, what is the value of $15\sin\theta + 20\cos\theta$?

12. Keisha has some 25¢, 26¢, and 35¢ stamps. She has a total of 180 stamps with a total value of $54.00. Keisha noticed that the numbers of stamps of each type form an arithmetic sequence. How many 35¢ stamps does Keisha have?

(A) 82 (B) 84 (C) 85 (D) 86 (E) 88

13. A standard deck of playing cards has 26 red and 26 black cards. Debbie and Don split a deck into two non-empty piles. there are four times as many black cards as red cards. In the number of red cards is an integer multiple of the number of black cards.

19. Suppose L and M are positive real numbers for which Ž ‘ %L L Ž ‘ %ML Ž ‘ %: L E M.

Compute the value of ã/ä.

(A) ¾6/¾9 (B) 6/¾9 (C) (¾9?5)/8 (D) (¾9?5)/6 (E) ¾9>

9. C Let N be the number of red marbles and > be the number of blue marbles. Before drawing a marble, we know that the probability of drawing a blue marble is $\frac{Õ}{Õ> å} = \frac{5}{8}$. After drawing one marble, the probability of drawing a blue marble has decreased. Thus, a blue marble was drawn, and now the probability of drawing a blue marble is $\frac{Õ? 5}{Õ? 5> å} = \frac{5}{9}$. We can rewrite these two equations as v> L > E N and w> F w L > s E N. Solving these two equations, we obtain 1> = 4 and 1N = 12, and hence there are 12 red marbles in the bag.

10. D We are given that 0 L :T; E :T E s; E :T E t ; L uT E uá and also that 1 0 1 L :U; E :U E s; E :U E t ; E :U E u; L vU E x for some positive integers x and y. Therefore, vU L u:T s;, which is satisfied for any value of y that is a multiple of 3. Since vU E x 1Q t r t s, U Q w r ä y w Then the possible values for y 501, for a total of 167 possibilities.

11. C Since each row has a product of 2021, the product . 1) # 4 + 6 * / has a value of t r t s 7. But the product) + / = 2021, and the product 4 6 = 47 (since) 4 6 L t r t s L v u® v y and we are given) = 43). Then (* 1 . #)(4 6)( ) + /) = (* 1 . #)(47)(2021) = t r t s 7 and * 1 . # = 86903.

12. B Let x = the number of 25 cent stamps and y = the number 711 of 35 cent stamps. Then 180 x 1y = the number of 26 cent stamps. Therefore, q0.0000080912 0 612 792 reW*n2(The)5(r)-6(e)4(for)-3(e)4(TQq0.00000912 0

15. C For 4 heads to appear before 2 tails, there can be 4 heads in a row or 4 heads out of 5 tosses. P(4 heads) = $\left(\frac{5}{6}\right)^{8} = 5$

20. C <u>Method 1</u>: Triangle ADP has area ½ (3)(2) = 3 and hypotenuse equal to ¾s u Since BAQ ADP, BQA APD. Then, AQO APD with scale factor $\frac{6}{¾5\ 7}$, Thus, AOQ is a right angle.

Then _ p c m d p g _ l e j c

23. C Let $y = \frac{2x-7}{2x^2-2x-5}$. Then $(2y)x^2 + (-2-2y)x + (7-5y) = 0$. This is a quadratic equation in x. Use the quadratic formula

$$x = \frac{2y+2 \pm \sqrt{(-2-2y)^2 - 4(2y)(7-5y)}}{4y}.$$

For x to be a real number, the discriminant must be non-negative.

$(-2-2y)^2 - 4(2y)(7-5y) \geq 0 \qquad 4(11y-1)(y-1) \geq 0.$

Then $y \leq \frac{1}{11}$ or $y \geq 1$, and there are no attainable values of y in the interval $\frac{1}{11} < y < 1$.

Thus, the least value of y is $\frac{1}{11}$.

24. A Method 1

Let $A_n = x^n - \frac{1}{x^n}$, and note that for $n = 1$,

$$(A_1)^2 + 3 = \left(x - \frac{1}{x}\right)^2 + 3 = \left(i\sqrt{2}\right)^2 + 3 = 1$$

Also, $(A_n)^3 = x^{3n} - 3x^n + \frac{3}{x^n} - \frac{1}{x^{3n}} = A_{3n} - 3A_n$.

Therefore, $A_{3n} = (A_n)^3 + 3A_n = A_n[(A_n)^2 + 3]$.

Hence, $A_3 = A_1[(A_1)^2 + 3] = A_1 = i\sqrt{2}$

Method 2

$x - \frac{1}{x} = i\sqrt{2} \qquad x^2 - 1 = x(i\sqrt{2})$. Squaring both sides and simplifying, $x = \pm\sqrt[4]{-1}$

Use De

$x = \pm(\cos 180^\circ + i\sin 180^\circ)^{\frac{1}{4}} = \pm(\cos 45^\circ + i\sin 45^\circ) = \pm\frac{\sqrt{2}}{2}(1+i)$

Factoring $x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right)$

For both possible values of x, $x^2 = \left(\frac{\sqrt{2}}{2}\right)^2 (1+i)^2 = i$ and $\frac{1}{x^2} = \frac{1}{i} = -i$

Therefore, $x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right) = i\sqrt{2}(i + 1 - i) = i\sqrt{2}$.

25. A Method 1: Construct the chords [illegible] and [illegible] and note that [illegible] because they intercept congruent arcs.

Using the Law of Cosines on △ABD,

$$(BD)^2 = 4^2 + 11^2 - 2(4)(11)\cos 60^\circ = 93.$$

Finally, constructing radii $\overline{\quad}$ and $\overline{\quad}$ and noting that m∠APD = 120, use the Law of Cosines on △APD.

$$93 = r^2 + r^2 - 2(r)(r)\cos 120^\circ = 3r^2 \qquad r^2 = 31$$

Therefore, the area of circle P is 31π.

Method 2: Construct $\overline{\quad}$ and